

CHARACTERIZATION OF THE SPATIAL VARIABILITY OF CHANNEL MORPHOLOGY

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ABSTRACT

The spatial variability of two fundamental morphological variables is investigated for rivers having a wide range of discharge (five orders of magnitude). The variables, water-surface width and average depth, were measured at 58 to 888 equally spaced cross-sections in channel links (river reaches between major tributaries). These measurements provide data to characterize the two-dimensional structure of a channel link which is the fundamental unit of a channel network.

The morphological variables have nearly log-normal probability distributions. A general relation was determined which relates the means of the log-transformed variables to the logarithm of discharge similar to previously published downstream hydraulic geometry relations. The spatial variability of the variables is described by two properties: (1) the coefficient of variation which was nearly constant (0.13–0.42) over a wide range of discharge; and (2) the integral length scale in the downstream direction which was approximately equal to one to two mean channel widths. The joint probability distribution of the morphological variables in the downstream direction was modelled as a first-order, bivariate autoregressive process. This model accounted for up to 76 per cent of the total variance. The two-dimensional morphological variables can be scaled such that the channel width–depth process is independent of discharge. The scaling properties will be valuable to modellers of both basin and channel dynamics. Published in 2002 John Wiley & Sons, Ltd.

KEY WORDS: channel; morphology; spatial variability; hydraulic geometry; scaling

INTRODUCTION

Two characteristics (length and change in elevation) of channel links, defined to be channel reaches between two adjacent junctions in a network (Shreve, 1966), have been investigated thoroughly by Horton (1945), Shreve (1967, 1969), Gupta and Waymire (1989), and Tarboton *et al.* (1989). However, the statistical characteristics of the variability of channel width and depth within these links have received less attention. Variability of width and depth can be conceptualized as a wave spectrum in which deterministic processes are characterized by individual wavelengths (Speight, 1965; Chang and Toebes, 1970; Church, 1972; Thornes, 1976a,b) and stochastic processes are characterized by bands of wavelengths (Ferguson, 1976; Howard and Hemberger, 1991). At the small-scale end of the spectrum, some investigators (Church, 1972; Thornes, 1976a,b; Furbish, 1985; Madej, 1999) found autocorrelations of width, slope, and thalweg in the downstream direction that suggests these stochastic processes have a ‘memory’ or a correlation distance. This correlation distance can be defined as the integral length scale analogous to the integral time scale used to characterize turbulence (Batchelor, 1982; Hinze, 1975).

Characterizing spatial variability depends upon the nature of the problem. Interpreting the variation of channel geometry (Wolman, 1955; Knighton, 1975; Leopold and Wolman, 1957; Speight, 1965; Ferguson, 1975, 1976), evaluating aquatic habitats (Mathur *et al.*, 1985; Bren, 1993; Johnson, 1994; Currier, 1995; Myers and Swanson, 1997), and studying the dispersion of tracers (Hays *et al.*, 1966; Sabol and Nordin, 1978; Nordin and Troutman, 1980; Bencala *et al.*, 1993; D’Angelo *et al.*, 1993; Runkel and Bencala, 1995)

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require knowledge of the marginal probability distribution of widths and depths, while flood (Fread, 1985; Rashid and Chaudhry, 1995; Wiele and Smith, 1996) and sediment routing models (Pickup and Higgins, 1979; Miller, 1994; Dietrich and Whiting, 1989) need both the marginal probability distributions and the proper length scales over which relatively small-scale variability can be suitably averaged to provide useful physical parameters for modelling.

Most geomorphic studies of stream channels have not been designed to address small-scale spatial variability. Hydraulic geometry has been measured at a few irregularly spaced cross-sections (Leopold and Maddock, 1953; Wolman, 1955; Andrews, 1979) and therefore cannot be used to determine statistical or spectral characteristics. Planform studies based on aerial photographs (Speight, 1965; Chang and Toebe, 1970; Ferguson, 1975; Dury, 1984; Howard and Hemberger, 1991) can provide the necessary samples and regular spacing but cannot determine characteristics of depth. Some regularly spaced measurements have been made on a few ephemeral Arctic rivers (Church, 1972) and on a few Spanish rivers (Thornes, 1976a,b). In both cases the range of discharges was about one order of magnitude. The focus of this study was to investigate the character of the spatial variability of channel morphology at the channel link-scale over a much wider range of discharges. Our goal was to provide statistical characterization of widths and depths within a channel link so that three-dimensional models of channels could be created by building upon the one-dimensional work. Thus, the specific purposes were: (1) to determine the character of the probability distributions of surface-water width and average depth and their relations to discharge; (2) to develop a suitable model of spatial variability in channels; and (3) to determine the integral length scale.

FIELD SITES

In order to obtain measurements spanning five to six orders of magnitude of discharge, measurements had to be collected from rivers of different size and in some cases for multiple discharges in the same river. Each river reach was a channel link with no major tributaries so that the discharge was constant. The small river was Clear Creek, which is a partially incised mountain stream on the eastern slope of the Front Range of Colorado (Figure 1). The reach was 920 m between Mill Creek and Fall River (Table I). It is essentially straight with no step-pool features, bed material of cobbles and boulders ($D_{50} = 180$ mm; Jarrett, 1984), and banks consisting of gravel, boulders, and alluvium. The medium river was Powder River which is a high-plains meandering river bordered by badlands and draining northeastern Wyoming and southeastern Montana (Hembree *et al.*, 1952). The reach was 4860 m between Clear Creek in Wyoming and Little Powder River in Montana (Moody *et al.*, 1999). This reach has pool-riffle features that are clearly present at discharges less than about $6 \text{ m}^3 \text{ s}^{-1}$. The bed material is primarily sand ($D_{50} = 0.25$ mm) with about 2 per cent gravel (Hembree *et al.*, 1952). The banks are formed by low floodplains or terraces of various heights and are composed of silt and sand (Leopold and Miller, 1954). The large river was the Mississippi River and, even though attempts have been made to control flooding with artificial levees, much of the river flows through a natural channel formed by the river. Two reaches were selected. One reach on the Upper Mississippi River was 48 000 m between the Meramec and Kaskaskia Rivers. Bed material is predominantly medium sand ($D_{50} = 0.38$ mm; Moody and Meade, 1992) and banks are composed of sand but intersected by lateral rock dykes spaced about 500 m apart. The other reach was 571 000 m on the Lower Mississippi River between the mouth of the Ohio River (954 river miles upstream from the Gulf of Mexico) and the mouth of the White River (600 river miles upstream from the Gulf of Mexico). Bed material is predominantly medium and coarse sand ($D_{50} = 0.57$ mm; Moody and Meade, 1993a,b), and banks are alluvial but often protected by revetment. The Lower Mississippi River has multiple channels in places, while the other three rivers have a single channel.

METHODS

Morphological variables were measured and calculated within the reaches described above at equally spaced cross-sections located about one mean-channel width apart (Table I). The basic cross-sectional field data were two-dimensional and consisted of horizontal distances measured from the left bank and water depths. For the Lower Mississippi River, measurements were collected from a digital elevation model based on hydrographic

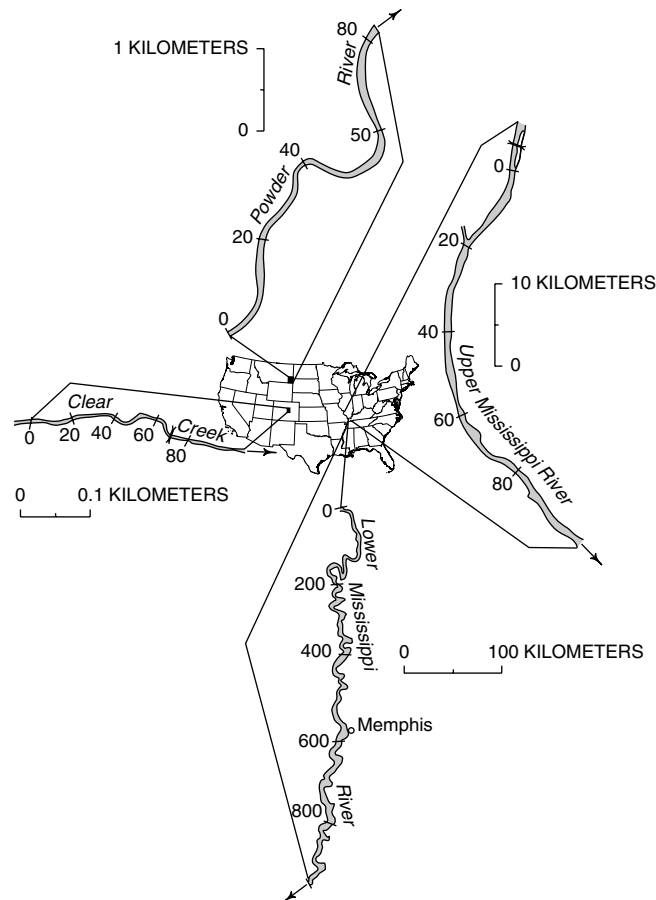


Figure 1. Map showing the location of the four study reaches. The numbers along the channel correspond to the number of the cross-section. The channels are drawn at four different scales

surveys for 1988–89. Depths were measured from 3.3 m (10 feet) above the low-water reference plane which corresponded to discharges of $8800 \text{ m}^3 \text{ s}^{-1}$ at Hickman, Kentucky (River Mile 920); $10\,900 \text{ m}^3 \text{ s}^{-1}$ at Memphis, Tennessee (River Mile 735); and $11\,800 \text{ m}^3 \text{ s}^{-1}$ at Helena, Arkansas (River Mile 663).

The spatial variability of two morphological variables (water-surface width, W , and average depth, D ; Figure 2) was characterized using several statistical methods. First, quantile–quantile plots of the raw data and of \log_{10} -transformed data were examined to assess adequacy of a normality assumption. The logarithmic transform was found to yield nearly normal data and was then used throughout the analysis. Second, the mean of log-transformed variables was regressed with the logarithm of discharge to determine adherence to power-law scaling that is typically seen in channel geometry studies. Third, changes in the coefficient of variation with discharge are examined to determine whether the distributions obey a simple scaling structure. Fourth, a bivariate autoregressive model was fit in order to examine the integral length scales of width and depth. Variation of the integral length scale with discharge was again examined using logarithmic regression.

Trends

The only adjustment for trends (or non-stationarity) that was done was to divide the data for the Lower Mississippi River into three subreaches. This non-stationarity was not the result of a continuous trend but rather conceived as two discontinuities dividing the three subreaches. The approximate locations of the discontinuities for the Lower Mississippi River were at River Miles 900 and 840. These were proposed by

Table I. Spatial sampling characteristics of the four channels

Characteristics	Clear Creek	Powder River		Upper Mississippi River	Lower Mississippi River
Approximate discharge during sampling ($\text{m}^3 \text{s}^{-1}$)	0.9	1.5	17	4620 ^b	8800 ^c
Annual mean discharge ($10^9 \text{ m}^3 \text{a}^{-1}$) ^a	0.13	0.4	0.4	165	10 900 ^d 11 800 ^e 400 ^f
Average slope	0.015	0.0015	0.0015	0.000096	0.000088 ^g
Length of study reach (m)	920	1980	3420	48 000	571 200
Sinuosity	1.02	1.20	1.20	1.06	1–3 ^h
Number of cross-sections	93	67	58	97	888
Average number of depth measurements	17	22	19	55	variable ⁱ
Sampling interval between cross-sections (m)	10	30	60	500	644
Mean channel width (m)	11.4	33.5	48.5	576	1150
Length of study reach (mean channel widths)	81	59	71	83	497

^a Mean annual discharges, US Geological Survey (1989, 1997).

^b Upper Mississippi River Mile 167.6 to 136.2 (river miles decrease downstream).

^c Approximately from Lower Mississippi River Mile 954 to 900 (0–76 mean channel widths).

^d Approximately from Lower Mississippi River Mile 900 to 840 (76–160 mean channel widths).

^e Approximately from Lower Mississippi River Mile 840 to 600 (160–495 mean channel widths).

^f Moody and Meade (1993a).

^g US Army Corps of Engineers, Memphis (1995).

^h Schumm *et al.*, (1994).

ⁱ Depth measurements were 15 to 30 m apart (S. Cobb, pers. comm., 1998).

Schumm *et al.* (1994) based on lithology and faulting with each subreach having significantly different river slopes. Reaches A, B, and C, correspond to above the New Madrid Bend (River Mile 954 to 900), the New Madrid Bend (River Mile 900 to 840), and below the New Madrid Bend (River Mile 840 to 600). Some experimentation was done with polynomial trend fitting in order to account for some of the minor trends such as the slight widening of the Upper Mississippi River in the downstream direction. However, it was felt that such a procedure would generally have been arbitrary, without physical justification, and would have addressed only secondary effects. Therefore, it was better to let the autoregressive model account for these minor trends.

Autoregressive model

The spatial dependence in the data was analysed by fitting a bivariate order-one autoregressive model to the mean-corrected width and depth series. Such a model provides a relation for predicting successive values of surface width and average depth in the downstream direction. Because a logarithmic transformation rendered the data more nearly normal in most cases, the autoregressive model was fitted using transformed data. Letting W_n and D_n be the water-surface width and the average depth, respectively, at cross-section n ,

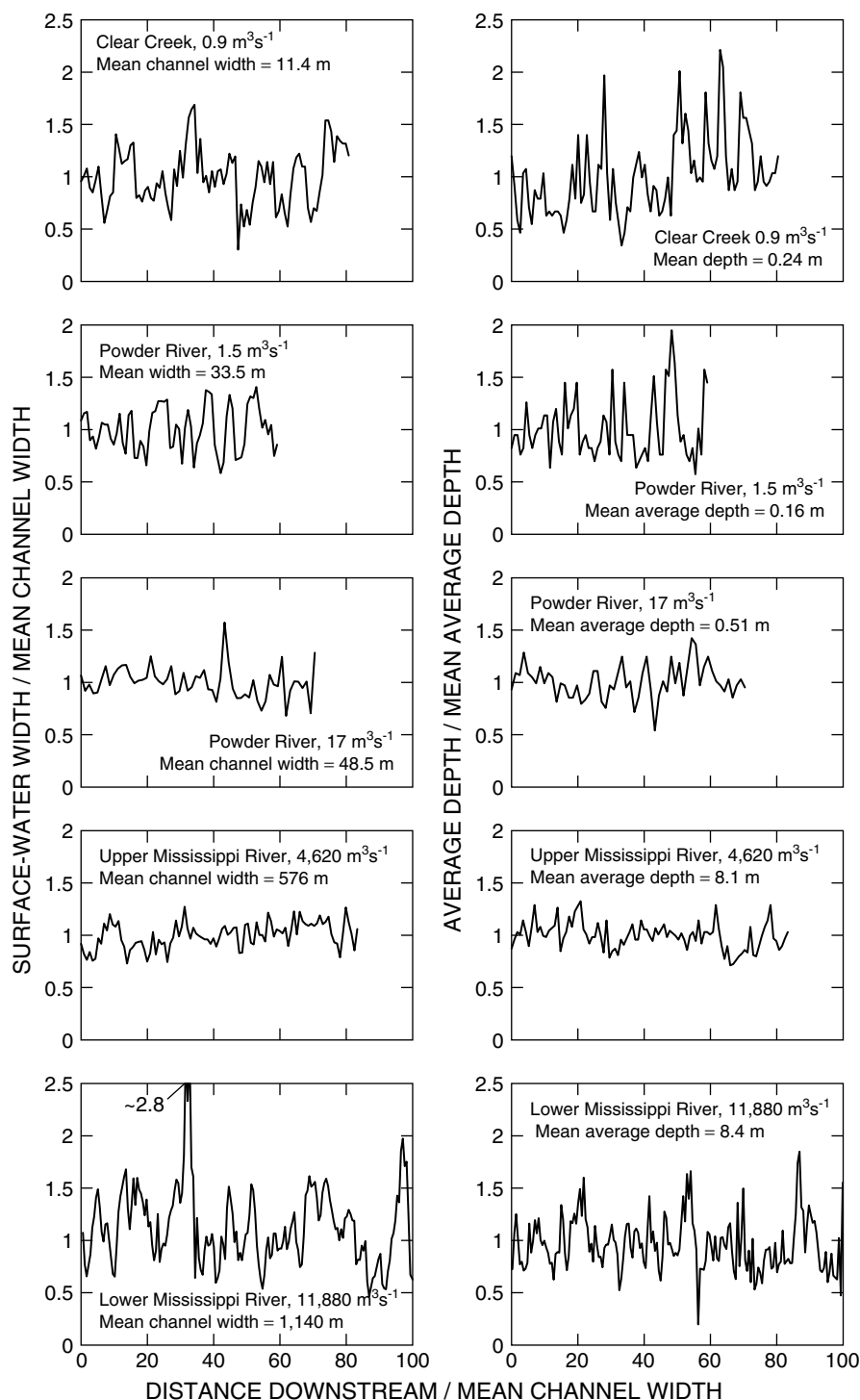


Figure 2. Plots of the basic data of water-surface widths and average depths. Plotted on the left-hand side are water-surface widths normalized by the mean channel width for each river. Plotted on the right-hand side are average depths normalized by the mean average depth for each river. Plots on the right-hand side correspond to the same locations as noted on the left-hand side

then the bivariate process is defined by:

$$\mathbf{Z}_n = \begin{bmatrix} Z_{1n} \\ Z_{2n} \end{bmatrix} = \begin{bmatrix} \log W_n - \mu_W \\ \log D_n - \mu_D \end{bmatrix} \quad (1)$$

where μ_W and μ_D are the means of the log-transformed variables. Thus, \mathbf{Z}_n is a vector of length two. The autoregressive model for the process \mathbf{Z}_n is:

$$\mathbf{Z}_n = \phi \mathbf{Z}_{n-1} + \mathbf{A}_n \quad (2)$$

In this equation, \mathbf{Z}_n , measured at cross-section n , is expressed in terms of \mathbf{Z}_{n-1} , which is measured at cross-section $n - 1$ immediately upstream from cross-section n . The matrix

$$\phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$$

is estimated from the data; its magnitude gives a measure of the degree of dependence in the variables from cross-section to cross-section. \mathbf{A}_n is an independent bivariate random series representing the unexplained variability in the model.

In order to estimate the integral length scales, it is further assumed that the cross-sectional data \mathbf{Z}_n were measurements of a continuous spatial process:

$$\mathbf{Y}(\mathbf{x}) = \begin{bmatrix} Y_1(x) \\ Y_2(x) \end{bmatrix} \quad (3)$$

where $Y_1(x) = \log W(x) - \mu_w$ and $Y_2(x) = \log D(x) - \mu_D$, letting $W(x)$ and $D(x)$ be the water-surface width and average depth at location x , which is a continuous variable. This continuous process is sampled at points Δx apart, or $\mathbf{Z}_n = \mathbf{Y}(n\Delta x)$. It is the underlying process $\mathbf{Y}(x)$ about which we are interested in making inferences. Assuming the existence of the continuous process $\mathbf{Y}(x)$ allows us to obtain results that are not dependent on the magnitude of the sampling interval Δx . This is important because Δx changes from river to river. Assume that the bivariate process $\mathbf{Y}(x)$ obeys a continuous analogue of Equation 2, namely:

$$\frac{d}{dx} \mathbf{Y}(x) + \theta \mathbf{Y}(x) = \mathbf{A}(x) \quad (4)$$

where θ is a 2×2 matrix of parameters. It may be shown that if a continuous process obeying Equation 4 is sampled at equal intervals Δx , then the resulting process obeys the model Equation 2, provided the parameters of the models are related by:

$$\phi = e^{-\theta \Delta x}. \quad (5)$$

The exponential of a matrix in this equation is interpreted in the usual way, i.e. in terms of a series expansion, or $\exp(M) = \sum_{i=0}^{\infty} M^i / i!$ where M is a matrix.

Integral length scale

In this paper, the integral length scale is the same as the correlation distance defined by others. The integral length scales L_i ($i = 1$ or 2) for the two components of the bivariate process $\mathbf{Y}(x)$ are defined to be analogous to the integral time scale (Batchelor, 1982) to be:

$$L_i = \int_0^{\infty} R_i(u) du, \quad (6)$$

where $R_i(u)$ is the autocorrelation function of $Y_i(x)$. The covariance function $C(u)$ of $Y(x)$ at lag u is given by:

$$C(u) = \exp(-\theta u)C(0) \quad (7)$$

(Jenkins and Watts, 1968, p. 474), from which we may obtain the autocorrelation function by $R(u) = \Gamma^{-1}C(u)\Gamma^{-1}$ where Γ is a diagonal matrix with diagonal elements equal to the standard deviations $\sqrt{c_{ii}(0)}$. From Equation 6 with covariance function given by Equation 7, it is seen that the integral length scales are diagonal elements of the 2×2 matrix:

$$\Gamma^{-1}\theta^{-1}C(0)\Gamma^{-1} = \Gamma^{-1}\theta^{-1}\Gamma R(0) \quad (8)$$

The computations in obtaining the integral length scale, given parameter estimates obtained by analysis of the discrete data, are most easily done as follows. Let γ_i be the eigenvalues of θ , $U = (u_1, u_2)$ the matrix of eigenvectors, and

$$V = U^{-1} = \begin{bmatrix} v_1^T \\ v_2^T \end{bmatrix}$$

where T denotes transpose. Then using Equation 7 and standard results in matrix algebra (Searle, 1982, chapter 11), the covariance matrix $C(u)$ may be written as:

$$C(u) = (\alpha e^{-\gamma_1 u} + \beta e^{-\gamma_2 u})C(0) \quad (9)$$

where $\alpha = u_1 v_1^T$ and $\beta = u_2 v_2^T$. Moreover, from Equation 5 the eigenvalues λ_i of ϕ are related to those of θ by

$$\lambda_i = e^{-\gamma_i \Delta x} \text{ or } \gamma_i = -\frac{\log \lambda_i}{\Delta x}$$

and the two matrices have the same eigenvectors. Therefore, the integral length scales for the bivariate process are obtained as diagonal elements of a 2×2 matrix:

$$(L_1, L_2) = \text{diag} \left[\Gamma^{-1} \left(\alpha \frac{-\Delta x}{\log \lambda_1} + \beta \frac{-\Delta x}{\log \lambda_2} \right) \Gamma R(0) \right] \quad (10)$$

Estimates from the data were obtained by substituting for λ_i in Equation 10, the eigenvalues of the estimated autoregressive parameter ϕ . Likewise the corresponding eigenvectors were used to obtain estimates for α and β . $C(0)$ (and hence Γ) are simply estimated using corresponding second moments of the data.

It is important to appreciate the implications of assuming the existence of the continuous process $Y(x)$ obeying Equation 4. In doing this it is assumed that the exponential form of the covariance function in Equation 7 is known to hold for all lags u . In fact, the covariance function can be estimated only for lags that are a multiple of the sampling interval, so assumption of the form of the continuous process $Y(x)$ actually represents an extrapolation of what behaviour the variables would exhibit if they had been sampled at shorter lags. This phenomenon can be viewed in terms of the spectrum, which is the Fourier transform of the correlation function. The continuous autoregressive process in Equation 4 has a spectrum that is defined for all wave numbers, whereas the discrete process Z_n possesses a spectrum only for wave numbers with magnitude less than $1/2\Delta x$, the Nyquist wave number (spatial analogue of the Nyquist frequency). Determination of the integral length scale based on the correlogram of a discrete process is dependent on the sampling interval. By assuming the continuous mode, the method is independent of the sampling interval and we can extrapolate to wave numbers beyond the Nyquist wave number. However, the caveat is that the goodness of the extrapolation depends on how well the process $Y(x)$ actually obeys the assumed model (Equation 4).

RESULTS

The probability distribution of water-surface width and average depth are nearly log-normal as indicated by quantile–quantile plots (Figure 3). The use of log transformations is consistent with methods that have been traditionally applied to estimate channel geometry (Leopold and Maddock, 1953), and thus, makes the data easy to compare with previous work. The most noticeable departures from normality are for one or two data points in the tail of the distribution in some cases (for example: $\log(\text{Width})$ and $\log(\text{Depth})$ in the Powder River, $17 \text{ m}^3 \text{ s}^{-1}$).

We next look in more detail at results for: (1) the mean of the probability distribution of log-transformed variables; (2) the standard deviation of the distribution expressed as the coefficient of variation; (3) joint probability distribution of water-surface widths and average depths represented by the autoregressive model; and (4) the downstream correlation distance or the integral length scale. We are especially interested in variations in the properties as a function of discharge. Discharge may be thought of as a measure of scale, and we thus wish to look at scaling properties of the probability distributions of the variable. It is necessary to understand these scaling properties if, for example, extrapolations are to be made to other rivers for which detailed morphological data may not be available or for modelling purposes.

Mean of the probability distributions

The mean of the log-transformed variables is found to be linearly related to the logarithm of discharge. Regression relations are:

$$\hat{\mu}_W = \log(17Q^{0.45}) \quad r^2 = 0.98 \quad (11)$$

$$\hat{\mu}_D = \log(0.18Q^{0.43}) \quad r^2 = 0.99 \quad (12)$$

where $\hat{\mu}$ is the regression estimated values of the mean of the log-transformed variables. These linear relations in the transformed variables are equivalent to power laws for untransformed variables and are identical in form to those proposed by Leopold and Maddock (1953) for downstream variations in width, depth, and velocity in the same river or drainage system.

Coefficient of variation

The coefficient of variation provides a measure of spatial variability relative to the mean. It varied in a relatively narrow range from 0.13 to 0.42 over four orders of magnitude in discharge (Table II). The minimum value was essentially the same (0.13–0.17) for both the Powder River ($17 \text{ m}^3 \text{ s}^{-1}$) and the Upper Mississippi River. The maximum values (0.38–0.42) were for the Lower Mississippi River. Average values of the coefficient of variation for the two morphological variables were 0.25 for W and 0.28 for D .

Autoregressive model

The autoregressive model was fit initially by allowing the order to be greater than one. For a more general order $p > 1$ model, the right hand side of Equation 2 would be augmented by including terms in $Z_{n-2}, Z_{n-3}, \dots, Z_{n-p}$ in addition to Z_{n-1} . The Akaike information criterion (AIC) indicated that a first-order model ($p = 1$) was indicated in all cases except the Powder River ($1.5 \text{ m}^3 \text{ s}^{-1}$), the Lower Mississippi River (Reach B), and the Lower Mississippi River (Reach C), for which p was 2, 2, and 3, respectively. In each of these three cases, however, improvement over a first-order model was only marginal in terms of both the AIC value and the percentage explained variance. It was felt that applying the same model to all reaches would be advantageous in terms of making comparisons. In addition, the straightforward correspondence between discrete and continuous models, as in Equations 2 and 4, does not hold if $p > 1$. Thus, all cases were reanalysed with $p = 1$. Estimates of the autoregressive parameters ϕ and the corresponding percentage of total variance explained by this model are given in Table II.

Integral length scale

The integral length scales, L_i , computed using Equation 10, are given in Table II. They were found to exhibit power-law scaling with discharge, and this scaling was not significantly different (at the 5 per cent

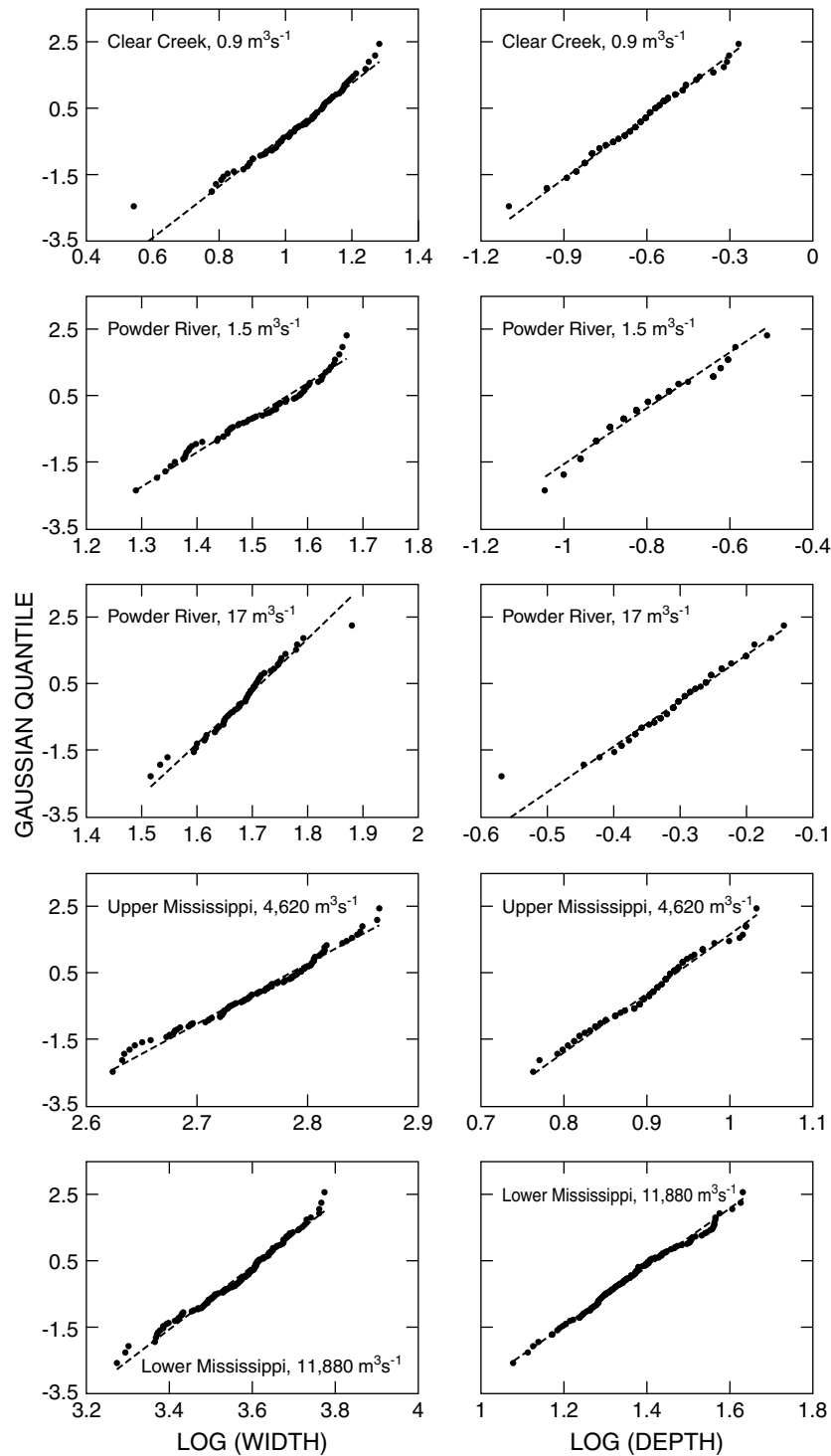


Figure 3. Quantile–quantile probability plots. Plotted on the left-hand side are water-surface widths and plotted on the right-hand side are average depths. Plots correspond to the same locations as noted in Figure 2

Table II. Statistical characteristics of populations of cross-section morphological variables sampled at regularly spaced channel cross-sections

Cross-section morphological variables ^a	Mean	Mean	Coefficient of variation	Autoregressive model			Integral length-scale L	
	Log_{10} -transformed variable			$\begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$	Percentage of total variance	(meters)	(mean channel widths)	
Clear Creek, 0.9 m ³ s ⁻¹								
W (m)	11.4	1.04	0.30	0.62	0.12	32	14	1.2
D (m)	0.24	-0.64	0.37	-0.29	0.48	38	15	1.3
Powder River, 1.5 m ³ s ⁻¹								
W (m)	33.5	1.51	0.22	0.85	0.41	38	36	1.1
D (m)	0.16	-0.82	0.27	-0.58	-0.05	19	19	0.6
Powder River, 17 m ³ s ⁻¹								
W (m)	48.5	1.68	0.15	0.16	0.00	3	32	0.7
D (m)	0.51	-0.30	0.17	-0.19	0.21	11	49	1.0
Upper Mississippi River, 4620 m ³ s ⁻¹								
W (m)	576	2.76	0.13	0.45	-0.14	29	830	1.4
D (m)	8.1	0.91	0.13	-0.19	0.41	27	780	1.4
Lower Mississippi River, reach A, 8800 m ³ s ⁻¹								
W (m)	1160	3.05	0.24	0.82	0.09	55	1880	1.6
D (m)	7.3	0.85	0.26	-0.31	0.53	62	2040	1.8
Lower Mississippi River, reach B, 10900 m ³ s ⁻¹								
W (m)	1210	3.06	0.36	0.86	-0.01	76	4610	3.8
D (m)	10.6	0.99	0.42	-0.22	0.61	58	3100	2.6
Lower Mississippi River, reach C, 11800 m ³ s ⁻¹								
W (m)	1140	3.03	0.38	1.04	0.27	73	2920	2.6
D (m)	8.4	0.90	0.33	-0.47	0.31	69	2520	2.2

^a W , channel top width; D , average cross-sectional depth.

level) for water-surface width and average depth. Thus, the single regression equation is:

$$L = 14Q^{0.54} \quad r^2 = 0.97 \quad (13)$$

Examination of Table II suggests that the integral length scale and mean channel width are of comparable magnitude, so the behaviour of the ratio of integral length scale to mean channel width was considered. The ratio ranges from 0.7 to 3.8 for width and from 0.6 to 2.6 for depth (Table II). Log-linear regression with Q reveals that the ratio increases slowly as discharge increases:

$$\frac{L}{\bar{W}} = 0.86Q^{0.095} \quad r^2 = 0.95 \quad (14)$$

where \bar{W} is mean channel width.

DISCUSSION

Mean of the probability distributions

The regression relations 11 and 12 are estimates of the mean of the probability distribution of log-transformed water-surface width and average depth. It is of interest to compare these regressions relations to the relations obtained in downstream hydraulic geometry studies ($\hat{W} = aQ^b$ and $\hat{D} = cQ^f$) as defined by Leopold and Maddock (1953). Values they give for the exponents of Q are $b = 0.5$ and $f = 0.4$, which are

close to our values of 0.45 and 0.43 respectively. The fitted coefficients in downstream hydraulic geometry studies are usually obtained by log–log regression of width and depth with Q , so in this sense previous studies like this study yield estimates of the mean of log width and mean of log depth. The main difference is that, in previous studies to obtain downstream hydraulic geometry relations, discharge was typically fixed at mean annual discharge. In this study, discharge values were not so fixed. However, because the range of discharges under consideration in this study is large (1 to 10 000 m³ s⁻¹), at-a-station discharge variability is small compared to overall discharge variability, and the regression relations we obtained would have changed only slightly had we fixed our measurements at mean annual discharge. Thus, our relations 11 and 12 may be considered essentially downstream hydraulic geometry relations, although we have extended the concept of downstream hydraulic geometry from the way it is typically applied by pooling rivers that are from diverse geographical regions and that vary widely in discharge.

In obtaining Equations 11, 12, and 13 from the diverse rivers in this study, we were interested in exploring the possibility that there are universal scaling relations of width and depth with discharge. While we do not have the detailed cross-sectional data for rivers other than those in this study to look at integral length scale, we can draw upon existing data sets from around the world to look at the mean of the distributions. Pooling data from world rivers also helps alleviate the problem in traditional channel geometry studies of the large uncertainty in a and c that results from trying to determine these coefficients using only a narrow range (generally only one or two orders of magnitude) of values for W , D , and Q . We performed log–log regressions of W and D against Q for a subset of world rivers, ranging from small, steep mountainous streams to some of the largest alluvial rivers and for which discharges varied from 8×10^{-3} to 2×10^5 m³ s⁻¹. As stated above, at-a-station discharge variability is small compared to overall discharge variability in these regressions, so the discharge does not need to be fixed at mean annual discharge to obtain relations that are essentially downstream hydraulic geometry relations. The data for the subset of world rivers are plotted along with some of Leopold and Maddock's (1953) data and the data from this study (Figure 4). The regression relations obtained are:

$$\hat{W} = 7.2Q^{0.50 \pm 0.02} \quad (2.6 - 20.2) \quad (15)$$

$$\hat{D} = 0.27Q^{0.39 \pm 0.01} \quad (0.12 - 0.63) \quad (16)$$

where the 95 per cent confidence intervals for b and f follow the \pm sign and intervals for a and c are given in parentheses. The regression standard error of estimate for the general relations (15 and 16) are 0.22 and 0.18 respectively; because of the logarithmic transformation, these numbers represent coefficient of variation at a given discharge. Logarithmically transforming these equations would yield estimates of the mean of log width and mean of log depth. The estimate of the coefficients and exponents would probably change little with more data because the number of data pairs is 226 and these include measurements for the largest as well as some of the smallest rivers in the world.

In summary, there seems to be merit in the idea of looking at the width and depth scaling over a very wide range of discharges for rivers around the world, and the subset of rivers included in this study is representative in terms of the regression relations. These regressions, however, yield only the means of the logarithmically transformed width and depth; detailed cross-sectional data at regular intervals such as that in this study will be needed to test further the universality of power-law behaviour for the integral length scale.

Coefficient of variation

Generally, the coefficient of variation for data in this study tends to be fairly stable (0.13–0.42) over a wide range of discharge and thus, water-surface widths and average depths are on the average within about 40 per cent of the mean. The coefficient of variation for the Powder River (17 m³ s⁻¹) and for the Upper Mississippi River tended to be somewhat less than for the other channels. This is not surprising for the Upper Mississippi River which is engineered to be a uniform navigation channel but the Powder River seems to have achieved a similar uniform but self-formed channel within its relatively easily eroded floodplain. Clear Creek, on the other hand, had the largest variability which might be expected *a priori* because of the variability imposed by large, resistant and irregular bed material and topography. Similarly, a smaller mountain river

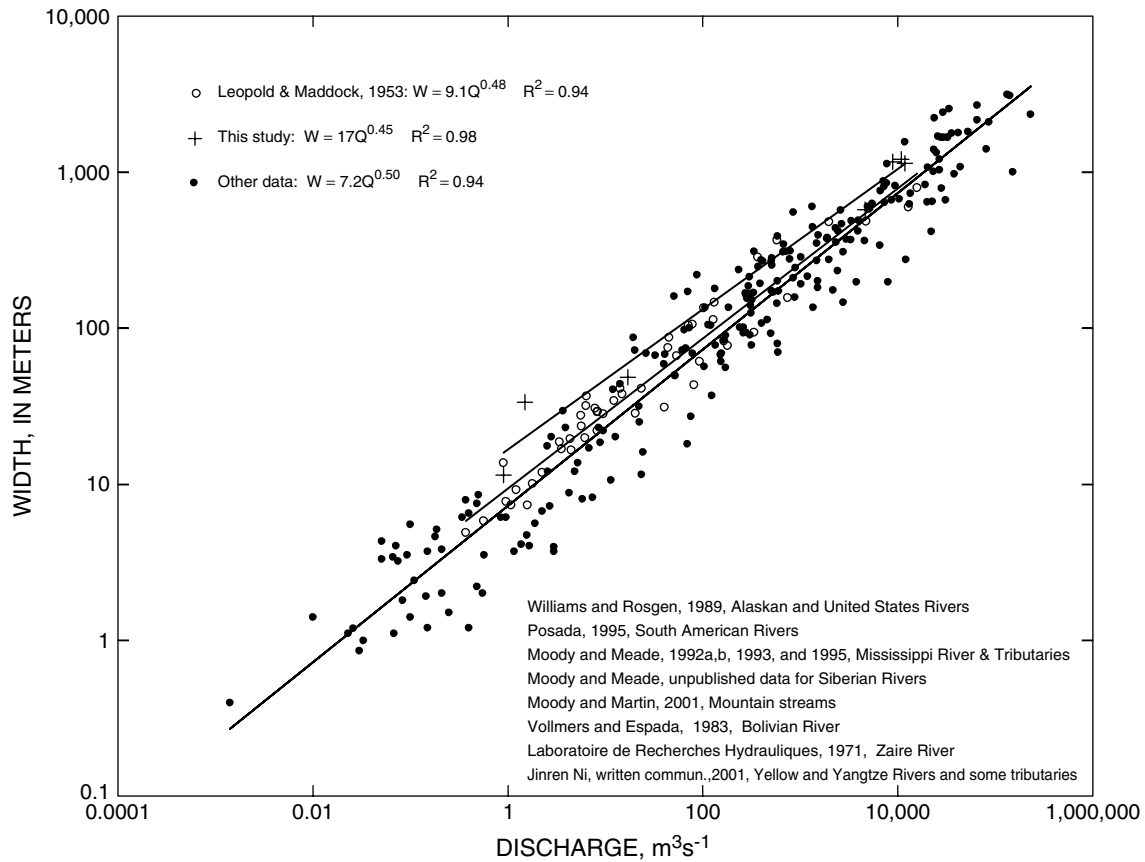


Figure 4. General relations between channel width and discharge. Data from this study are shown as + symbols, data from Leopold and Maddock (1953) are shown as open circles, and data from 226 measurements on various rivers throughout the world are shown as solid circles

(Middle Boulder Creek; Furbish, 1985) with mean annual discharge of $1.53 \text{ m}^3 \text{ s}^{-1}$ had an average coefficient of variation (0.45) slightly larger than Clear Creek. The increase in the coefficient of variation for the Lower Mississippi River may be a result of the multiple channels. Powder River is perhaps the most 'natural' river of the four studies in this paper, with no flood controls, no irrigation diversion structures, no urban impact, and no navigation, and had the lowest average variability (0.17). However, the average variability for the same river at lower discharge ($1.5 \text{ m}^3 \text{ s}^{-1}$) was greater (0.28) which supports the idea that higher discharges 'drown' out spatial variability present at lower discharges.

The small range of the coefficient of variation in these data is important for the following reason. Using the relations in Equations 11 and 12, the width and depth can be scaled by a function of discharge which makes the mean (with a logarithmic transformation) independent of discharge. In addition, if the changes in the coefficient of variation with discharge are small enough to be ignored, then the probability distribution (not just the mean) of these scaled morphological variables will be independent of discharge or scale. For example, using Equation 11, $W/17Q^{0.45}$ has a distribution that is independent of Q . If such a structure (known as 'simple scaling') holds, then estimates of the magnitude and variability of W may be made for channels for which detailed cross-sectional data are not available. This is similar to what is done for an index-flood approach to flood frequency analysis. For this approach, annual peaks are scaled by some function of discharge, such as the mean annual discharge, in order to obtain a quantity that has a distribution independent of drainage area. This allows scaled flows for different areas to be combined for regionalization. Gupta and Dawdy (1995) and Hosking and Wallis (1997) give a complete discussion of advantages and problems with such a procedure.

Integral length scale

For modelling purposes, the integral length scale is important because it determines how the correlation of width and depth changes with spacing of channel cross-sections in the downstream direction. The approach used in this study makes inferences about the underlying continuous bivariate spatial process in natural channels based on discrete measurements at intervals. Such an approach may be compared to studies for univariate processes done previously. The integral length scale given in Equation 10 for a bivariate continuous process reduces to $L_1 = -\Delta x / \log \phi_1$, where ϕ_1 is the lag one autoregression coefficient, for a discrete univariate process. Using this relation, integral length scales were estimated for discrete spatial series (supplied by M. Church) for the North and Middle Rivers on Baffin Island and for a steep alluvial cone channel, Harvey Creek, adjacent to Howe Sound, and for a small mountain stream in Colorado using ϕ_1 values reported by Furbish (1985). The estimates of the integral length scale are 2.8, 3.7, 0.2, and 1.1 mean channel widths and these fall within the range of values for this study. Madej (1999) reported correlation distances for thalweg depths in Redwood Creek were approximately one mean channel width and changed with time following a flood. Sidorchuk (1996) used spectral methods and identified unstable bed undulations on the order of two mean channel widths which were important in increasing bank erosion and meander development. These estimates of the integral length scale based on the discrete univariate process in the downstream direction, in the cross-stream, and in the vertical directions are very similar to the estimates of the integral length scale based on the assumption in this study of a continuous bivariate process.

The integral length scale, L , exhibits power law scaling with Q , and the coefficient and scaling exponent are the same for both morphometric variables. In the previous section it was noted that the distribution of morphological variables closely approximates a simple scaling structure. For example, $W/17Q^{0.45}$ has a distribution that is nearly independent of Q , where W is water-surface width at a single cross-section. With the additional information provided by the integral length scale, we can now determine the scaling structure of the entire stochastic processes defined by the morphometric variables as a function of longitudinal distance.

To be more specific, let us consider the channel width–depth process $\{W(x), D(x)\}$, where the brackets here indicate that we are interested in the joint probability distribution of these variables at multiple cross-sections. Any simulation of the width–depth process over a reach (as opposed to at a single cross-section) would require specifying this joint behaviour. Using the power-law relations in Equations 11, 12, and 13, it is seen that the scaled process:

$$\left\{ \frac{1}{17Q^{0.45}} W(14Q^{0.54}x), \frac{1}{0.18Q^{0.43}} D(14Q^{0.54}x) \right\} \quad (17)$$

has joint distributions that are independent of Q . This may be seen as follows. We may define a process $\{Y'(x)\}$:

$$Y'(x) = \begin{bmatrix} Y'_1(x) \\ Y'_2(x) \end{bmatrix} \quad (18)$$

where $Y'_i(x) = Y_i(14Q^{0.54}x)$. Thus $\{Y'(x)\}$ is $\{Y(x)\}$ in Equation 3 with a rescaling of downstream distance using discharge. The process $\{Y'(x)\}$ has covariance function given by $\exp(-14Q^{0.54}\theta u)C(0)$, using Equation 7. From Equations 8 and 13, the autoregressive parameter θ scales as $Q^{-0.54}$, i.e. the covariance function of $\{Y'(x)\}$ is independent of Q , and hence the scaled process in Equation 17 is independent of Q .

SUMMARY AND CONCLUSIONS

Spatial variability of two fundamental channel morphological variables, water-surface width and average depth, has been investigated at the channel-link scale where the discharge is nearly constant. The spatial variability within a channel link exhibits a similarity across large changes in scale (five orders of magnitude). These morphological variables have nearly log-normal probability distributions. General relations 11 and 12 were determined that relate the means of the log-normal probability distributions to the discharge. The coefficient of variation of the distributions varied little with changes in discharge and ranged from 0.13 to

0.42, indicating a simple scaling structure. A bivariate autoregressive model was used to investigate spatial correlation in the downstream direction and integral length scale. The integral length scale was found to exhibit a power-law scaling with discharge, Equation 13, and was generally about one to two mean channel widths in magnitude for both width and depth. In combination, these results indicate that scaling water-surface widths and average depths by the functions of discharge in Equations 11 and 12, and at the same time scaling downstream distance by the integral length-scale in Equation 13, yields processes that do not depend on discharge. Thus, the results are representative of a wide range of flow conditions and it is believed that, using the scaling behaviour, the results will be applicable to rivers not included in this study, although further detailed cross-sectional data for other rivers needs to be collected to test further this hypothesis.

One application of the bivariate autoregressive model developed in this study is to simulate width and depth at cross-sections or for channel links for which detailed data may not be available. This extends previous studies which have looked at statistical variability of link length and link elevation drop. The scaling properties investigated here will be valuable to modellers of both basin and channel dynamics. The solution accuracy and hence the predictive power of all types of models will improve if techniques can be developed to properly parameterize spatial variability of morphometric variables in these models.

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REFERENCES

- Andrews ED. 1979. Scour and Fill in a Stream Channel, East Fork River, Western Wyoming. US Geological Survey Professional Paper 1117.
- Batchelor GK. 1982. *The Theory of Homogeneous Turbulence*. Cambridge University Press: New York.
- Bencala KE, Duff JH, Harvey JW, Jackman AP, Triska FJ. 1993. Modelling within the stream-catchment continuum. In *Modelling Change in Environmental Systems*, Jakeman AJ, Beck MB, McAleer MJ (eds). John Wiley & Sons: Chichester; 163–187.
- Bren LJ. 1993. Riparian zone, stream, and floodplain issues: a review. *Journal of Hydrology* **150**: 277–299.
- Chang TP, Toebes GH. 1970. A statistical comparison of meander planform in the Wabash Basin. *Water Resources Research* **6**: 557–578.
- Church M. 1972. Baffin Island sandurs: A study of Arctic fluvial processes. Geological Survey of Canada, Bulletin 216.
- Currier PJ. 1995. Woody vegetation expansion and continuing declines in open channel habitat on the Platte River in Nebraska. Platte River Whooping Crane Maintenance Trust, Inc.
- D'Angelo DJ, Webster JR, Gregory SV, Meyer JL. 1993. Transient storage in Appalachian and Cascade mountain streams as related to hydraulic characteristics. *Journal of the North American Benthological Society* **12**(3): 223–235.
- Dietrich WE, Whiting P. 1989. Boundary shear stress and sediment transport in river meanders of sand and gravel. In *River Meandering*, Ikeda S, Parker G (eds). Water Resources Monograph 12. American Geophysical Union: 1–49.
- Dury GH. 1984. Abrupt variation in channel width along part of the River Severn, near Shrewsbury, Shropshire, England. *Earth Surface Processes and Landforms* **9**: 485–492.
- Ferguson RI. 1975. Meander irregularity and wavelength estimation. *Journal of Hydrology* **26**: 315–333.
- Ferguson RI. 1976. Disturbed periodic model for river meanders. *Earth Surface Processes* **1**: 337–347.
- Fread DL. 1985. Channel routing. In *Hydrological Forecasting*, Anderson MG, Burt TP (eds). John Wiley and Sons: Chichester; 437–503.
- Furbish DJ. 1985. *The stochastic structure of a high mountain stream*. PhD Thesis, University of Colorado.
- Gupta VK, Dawdy DR. 1995. Physical interpretations of regional variations in the scaling exponents of flood quantiles. *Hydrological Processes* **9**: 347–361.
- Gupta VK, Waymire E. 1989. Statistical self-similarity in river networks parameterized by elevation. *Water Resources Research* **25**: 463–476.

- Hays JR, Krenkel PA, Schnelle KB. 1966. *Mass transport mechanisms in open-channel flow*. Report Department of Civil Engg., Vanderbilt University, Nashville, Tenn.
- Hembree CH, Colby BR, Swenson HA, Davis JR. 1952. *Sedimentation and Chemical Quality of Water in the Powder River Drainage Basin Wyoming and Montana*. US Geological Survey Circular 170.
- Hinze JO. 1975. *Turbulence* (2nd edn). McGraw-Hill: New York.
- Horton RE. 1945. Erosional development of streams and their drainage basins; hydrophysical approach to quantitative morphology. *Geological Society of America Bulletin* **56**: 275–370.
- Hosking JRM, Wallis JR. 1997. *Regional Frequency Analysis*. Cambridge University Press: Cambridge.
- Howard AD, Hemberger AT. 1991. Multivariate characterization of meandering. *Geomorphology* **4**: 161–186.
- Jarrett RD. 1984. Hydraulics of high-gradient streams. *Journal of Hydraulic Engineering* **110**(11): 1519–1539.
- Jenkins GM, Watts DG. 1968. *Spectral Analysis and its Applications*. Holden-Day: San Francisco.
- Johnson WC. 1994. Woodland expansion in the Platte River, Nebraska: Patterns and causes. *Ecological Monograph* **64**: 45–84.
- Knighton AD. 1975. Variations in at-a-station hydraulic geometry. *American Journal of Science* **275**: 186–218.
- Laboratoire de Recherches Hydrauliques. 1971. *Etude de l'amelioration du bief maritime du Fleuve Zaire*. Rapport General, Mod. 255, Mateba VIII (in French).
- Leopold LB, Maddock T Jr. 1953. *The Hydraulic Geometry of Stream Channels and Some Physiographic Implications*. US Geological Survey Professional Paper 252.
- Leopold LB, Miller JP. 1954. *A Postglacial Chronology for Some Alluvial Valleys in Wyoming*. US Geological Survey Water-Supply Paper 1261.
- Leopold LB, Wolman MG. 1957. *River channel patterns: braided, meandering, and straight*. US Geological Survey Professional Paper 282-B, 39–85.
- Madej MA. 1999. Temporal and spatial variability in thalweg profiles of a gravel-bed river, *Earth Surface Processes and Landforms* **24**: 1153–1169.
- Mathur D, Bason WH, Purdy EJ Jr, Silver CA. 1985. A critique of the instream flow incremental methodology. *Canadian Journal of Fisheries and Aquatic Science* **42**: 825–830.
- Miller AJ. 1994. Debris-fan constrictions and flood hydraulics in river canyons: some implications from two-dimensional flow modelling. *Earth Surface Processes and Landforms* **19**: 681–697.
- Moody JA, Martin DA. 2001. *Hydrologic and sedimentologic response of two burned watersheds in Colorado*. US Geological Survey Water Resources-Investigation Report 01-4122.
- Moody JA, Meade RH. 1992. *Hydrological and sedimentologic data collected during three cruises at low water on the Mississippi River and some of its tributaries, July 1987–June 1988*. US Geological Survey Open-File Report 91-485.
- Moody JA, Meade RH. 1993a. *Hydrological and sedimentologic data collected during three cruises at low water on the Mississippi River and some of its tributaries, July 1987–June 1988*. US Geological Survey Open-File Report 91-485.
- Moody JA, Meade RH. 1993b. *Hydrological and sedimentologic data collected during four cruises at high water on the Mississippi River and some of its tributaries, March 1989–June 1990*. US Geological Survey Open-File Report 92-651.
- Moody JA, Meade RH. 1995. *Hydrological and sedimentologic data collected during three cruises on the Mississippi River and some of its tributaries from Minneapolis, Minnesota, to New Orleans, Louisiana, July 1991–May 1992*. US Geological Survey Open-File Report 94-474.
- Moody JA, Pizzuto JE, Meade RH. 1999. Ontogeny of a flood plain. *Geological Society of America Bulletin* **111**(2): 291–303.
- Myers TJ, Swanson S. 1997. Precision of channel width and pool area measurements. *Journal of the American Water Resources Association* **33**(3): 647–659.
- Nordin CF Jr, Troutman BM. 1980. Longitudinal dispersion in rivers: The persistence of skewness in observed data. *Water Resources Research* **16**: 123–128.
- Pickup G, Higgins RJ. 1979. Estimating sediment transport in a braided gravel channel—the Kawerong River, Bougainville, Papua New Guinea. *Journal of Hydrology* **40**: 283–297.
- Posada-Garcia I. 1995. *Transport of sands in deep rivers*. PhD Thesis, Colorado State University, Fort Collins, Colorado.
- Rashid RSMM, Chaudhry MH. 1995. Flood routing in channels with flood plains. *Journal of Hydrology* **171**: 75–91.
- Runkel RL, Bencala KE. 1995. Transport of reacting solutes in rivers and streams. In *Environmental Hydrology*, Singh VP (ed.). US Government: 137–164.
- Sabol GV, Nordin CF Jr. 1978. Dispersion in rivers as related to storage zones. *Journal of the Hydraulics Division, ASCE* **HY5**: 695–708.
- Schumm SA, Rutherford ID, Brooks J. 1994. Pre-cutoff morphology of the Lower Mississippi River. In *The Variability of Large Alluvial Rivers*, Schumm SA, Winkley BR (eds). American Society of Civil Engineers: 13–44.
- Searle SR. 1982. *Matrix Algebra Useful for Statistics*. John Wiley: New York.
- Shreve RL. 1966. Statistical law of stream numbers. *Journal of Geology* **74**: 17–37.
- Shreve RL. 1967. Infinite topologically random channel networks. *Journal of Geology* **75**: 178–186.
- Shreve RL. 1969. Stream lengths and basin areas in topologically random channel networks. *Journal of Geology* **77**: 397–414.
- Sidorchuk A. 1996. The structure of river bed relief. In *Coherent Flow Structures in Open Channels*, Ashworth PJ, Bennett SJ, Best JL, McLelland SJ (eds). John Wiley: Chichester: 397–421.
- Speight JG. 1965. Meander spectra of the Angabunga River. *Journal of Hydrology* **3**: 1–15.
- Tarboton DG, Bras RL, Rodriguez-Iturbe I. 1989. Scale and elevation in river networks. *Water Resources Research* **25**(9): 2037–2051.
- Thornes JB. 1976a. *Semi-arid Erosional Systems; Case Studies from Spain*. Department of Geography, London School of Economics and Political Science, Geographical Papers No. 7: 53–78.
- Thornes JB. 1976b. Autogeometry of semi-arid channel systems. In *Symposium on Inland Waterways for Navigation, Flood Control and Water Diversions, Vol. II*. Colorado State University, August 10–12, 1976. American Society of Civil Engineers: 1715–1725.
- Vollmers HJ, Espada GP. 1983. Sediment measurements in the Pilcomayo River in Bolivia. In *Proceedings of the Second International Symposium of River Sedimentation*. 11–16 October, 1983, Nanjing, China. Water Resources and Electric Power Press: 1050–1070.

- Wiele SM, Smith JD. 1996. A reach-averaged model of diurnal discharge wave propagation down the Colorado River through the Grand Canyon. *Water Resources Research* **32**(5): 1375–1386.
- Williams GP, Rosgen DL. 1989. *Measured total sediment loads (suspended loads and bedloads) for 93 United States streams*. US Geological Survey Open-File Report 89-67.
- Wolman MG. 1955. *The Natural Channel of Brandywine Creek Pennsylvania*. US Geological Survey Professional Paper 271.